Gyrators in electrical power management systems

St. Hărăguș*, D. Toader*, I. Tatai*

*Department of Physical Foundation of Engineering
“Politehnica” UniversityTimișoara,
Postal address: Timișoara, B-I V. Pârvan nr.2,
e-mail: stefan.haragus@et.upt.ro

Abstract – The paper analyses two main types of gyrator-like dc-dc switched converters encountered in dc power management. The first type consists of a reactive reciprocal two-port network inserted between two bridges of ideal switches. The power transfer from source to load is controlled by modifying the delay between the switching bridges. The second type is a buck converter with an appropriate sliding mode control, suitable for paralleling multiple converters.

Keywords: nonreciprocal circuits, gyrators, dc switching converters.

I. INTRODUCTION

The gyrator as an ideal, lossless nonreciprocal two-port circuit element has been introduced by the Dutch engineer B. D. H. Tellegen [1]. The main property of the gyrator, namely the conversion of the network connected at the output port into its dual, as seen from the input port, was supposed by Tellegen to help synthesizing various electrical filters in the field of signal processing.

There are many ways to physically implement a gyrator. The first gyrator device ever built was based on the Faraday rotation effect, and it operates in the range of microwaves [2]. In the quest for physical gyrators it has been discovered that the Hall effect can be used to build gyrators that could operate in a very broad range of frequencies [3]. Unfortunately, Hall effect based gyrators do not imposed as inductance emulators due to their large ohmic dissipation.

Practical applications of the gyrator at low frequencies occurred with the development of active semiconductor device technology, especially due to the need to integrate large values of inductances. Based on transistors and operational amplifiers, many practical gyrator circuits have been proposed and used [4].

All the gyrator realizations mentioned so far were used in the signal processing technique and operates time-continuously. On the other hand, in power management some dc-dc switching converters that have a gyrator-like behavior are being used. Thus, a switched double-bridge dc-dc converter behaves as a dc gyrator, as long as the mean values of the voltages and currents at the ports are considered [5]. Another dc gyrator form the same class, based on a switched λ/4 transmission line, has also been reported [6]. A different class of gyrator-like dc-de converters makes use of the sliding-mode control [6, 7, 11]. In contrast to the double-bridge converter, which has a natural dc gyrator behavior, the second type of converters is forced to behave like a dc gyrator by a special control loop.

In this paper the dependence of two-port parameters of the double-bridge gyrator on the network’s parameters as well as on the switching delay between the bridges will be analyzed. Also, the configuring of a buck converter as a gyrator using sliding-mode control will be explained.

II. BASIC PROPERTIES OF ELECTRICAL GYRATORS

An antireciprocal two-port circuit is a special case of a nonreciprocal two-port circuit whose parameters satisfy a certain condition, usually referred as the antireciprocity condition. For example, if the two-port’s equations are expressed via the short-circuit admittances,

\[ Y_1 = \frac{U_1}{I_1} + Y_{12} \frac{U_2}{I_2}, \]

\[ Y_2 = \frac{U_2}{I_2} + Y_{21} \frac{U_1}{I_1}, \]

then the antireciprocity condition is [8]

\[ Y_{12} = -Y_{21}, \] (2)

where the passive rule for U and I at both ports has been assumed (Fig.1).

The physical meaning of eq.(2) is as follows: if a voltage source, connected at the input port, determines a current \( I \) at the short-circuited output port then, the same voltage source, if connected at the output port, will determine, at the short-circuited input port, a current of the same amplitude but of opposed phase, \( -I \). A similar antireciprocity condition results if the impedance formalism for the two-port is used:

\[ Z_{12} = -Z_{21}. \] (3)
For an ideal gyrator, which is a lossless passive antireciprocal two-port network, eqs. (1) become
\[ \begin{align*}
I_1 &= \pm gU_i, \\
I_2 &= \pm gU_i',
\end{align*} \tag{4} \]
where parameter \( g \) is given the name gyration conductance and is a constitutive parameter of the gyrator. The circuit symbol for a gyrator is shown in Fig. 1.

Eqs. (4) reveal the two basic properties of an ideal gyrator. Thus, if the gyrator is loaded by impedance \( Z_s \), then the input impedance is given by equation
\[ Z_{in} = r = \frac{1}{Z_s}, \tag{5} \]
where \( r = g' \) is the gyration resistance. This property allows emulating large inductances at low frequencies by using a capacitively loaded gyrator. On the other hand, from eqs. (4) also follows that
\[ P_1 = -P_2, \tag{6} \]
\( P_1 \) and \( P_2 \) being the active power at the input, respectively the output port of the gyrator. With the assumed passive rule at both ports, this means that the active power supplied at one port will be recovered at the other port. Hence, ideal gyrators belong to a larger power processing circuits called POPI (power output equals power input).

For power processing circuits not only the POPI feature is of relevance, but also is important to efficiently control the mean power transferred from one port to the other. In the case of gyrators as power management circuits, this is achieved by modifying the gyration conductance.

Switched gyrators are also described by two-port equations with the difference that the currents and voltages occurring in these equations represent the mean values of the instantaneous variables taken over a switching period. Hence, the two-port equations of a lossy switched gyrator can be expressed, for example, in the form
\[ \begin{align*}
I_1 &= G_{11}U_1 + G_{12}U_2, \\
I_2 &= G_{21}U_1 + G_{22}U_2.
\end{align*} \tag{7} \]
Therefore, switched gyrators can be viewed as dc gyrators.

An equivalent circuit can be associated to eqs. (7), as shown in Fig. 2.

**III. DC-DC CONVERTERS WITH NATURAL GYRATOR-LIKE BEHAVIOR**

Maybe the simplest switched gyrator is the double bridges d.c.-d.c. converter shown in Fig. 4.

The switching sequence for the input bridge is as follows: first, \( S_{11} \) is closed synchronously with \( S_{14} \) then, after a time interval of \( T/2 \), \( S_{12} \) and \( S_{13} \) are closed and, at the same moment, \( S_{11} \) and \( S_{14} \) are opened; after a time interval of \( T/2 \) the sequence starts again and so
on. The switches of the output bridge follows the same switching sequence as the input bridge, but delayed by a time interval \( T_D \). Further, we will assume that at the converter ports are connected two dc voltage sources, of magnitudes \( U_1 \), respectively \( U_2 \).

Therefore, assuming that the switches \( S_{11}, S_{12}, \ldots, S_{24} \) are ideal, the ports of the reciprocal two-port network are fed with rectangular voltages, \( u_a \) and \( u_b \), with magnitudes \( U_1 \), respectively \( U_2 \), delayed by a time interval \( T_D \) (Fig.5).

Our goal is to derive the dc two-port equations of this converter and the necessary conditions for gyrator-like behavior. The analysis, which is given in details in [9, 10] is based on the Fourier spectral decomposition of the rectangular voltage waveforms of the equivalent sources; this allows expressing the currents \( i_1(t) \) and \( i_2(t) \) in terms of the reciprocal two-port network parameters. Then, the currents \( i_1(t) \) and \( i_2(t) \) at the converter’s ports can be expressed in terms of \( i_a \) and \( i_b \), which eventually yields the mean currents \( I_1 \) and \( I_2 \) by integrating over a period \( T \):

\[
I_1 = G_{11}U_1 + G_{12}U_2 \quad \text{and} \quad I_2 = G_{21}U_1 + G_{22}U_2,
\]

where

\[
G_{11} = \frac{8}{\pi^2} \sum_{k=1,3,\ldots} \frac{1}{k^2} G_{11}^{(k)}, \quad G_{12} = \frac{8}{\pi^2} \sum_{k=1,3,\ldots} \frac{1}{k^2} G_{12}^{(k)},
\]

\[
G_{21} = \frac{8}{\pi^2} \sum_{k=1,3,\ldots} \frac{1}{k^2} (G_{12}^{(k)} \cos kT_D - B_{12}^{(k)} \sin kT_D), \quad G_{22} = \frac{8}{\pi^2} \sum_{k=1,3,\ldots} \frac{1}{k^2} G_{22}^{(k)},
\]

\[
G_{12}^{(k)} = \frac{8}{\pi} \sum_{k=1,3,\ldots} \frac{1}{k} (G_{12}^{(k)} \cos kT_D - B_{12}^{(k)} \sin kT_D), \quad G_{21}^{(k)} = \frac{8}{\pi} \sum_{k=1,3,\ldots} \frac{1}{k} G_{21}^{(k)} \cos kT_D + B_{21}^{(k)} \sin kT_D)
\]

The storage network being reciprocal, \( G_{12}^{(k)} = G_{21}^{(k)} \), \( B_{12}^{(k)} = B_{21}^{(k)} \), hence eqs.(11, 12) become

\[
G_{12} = \frac{8}{\pi^2} \sum_{k=1,3,\ldots} \frac{1}{k} G_{12}^{(k)} \cos kT_D - \frac{8}{\pi^2} \sum_{k=1,3,\ldots} \frac{1}{k^2} B_{12}^{(k)} \sin kT_D
\]

\[
G_{21} = \frac{8}{\pi^2} \sum_{k=1,3,\ldots} \frac{1}{k} G_{21}^{(k)} \cos kT_D + \frac{8}{\pi^2} \sum_{k=1,3,\ldots} \frac{1}{k^2} B_{21}^{(k)} \sin kT_D
\]

The dc two-port driving point conductances (eq.9, 10) do not dependent on the time delay \( T_D \). For a lossless reciprocal network \( G_{11}=G_{22}=0 \) and eqs.(8) correspond to a lossless nonreciprocal two-port:

\[
I_1 = G_{12}U_2 \quad \text{and} \quad I_2 = G_{21}U_1,
\]

with \( G_{12} \) and \( G_{21} \), given by eqs.(14, 15). These transfer conductances depend both on the nature of the reciprocal network elements, as well on the time delay \( T_D \). If the reciprocal network is composed solely of inductances and/or capacitances, then

\[
G_{11} = G_{22} = 0 \quad \text{and} \quad G_{12} = -\frac{8}{\pi^2} \sum_{k=1,3,\ldots} \frac{1}{k^2} B_{12}^{(k)} \sin kT_D = -G_{21}
\]

which corresponds to an ideal dc gyrator.

On the other hand, if \( T_D = T/4 \) then the converter behaves like a gyrator irrespective of elements of the reciprocal network. If \( T_D = 3T/4 \) then the dc transfer conductances swap signs. This is an important property of the switched gyrator as it can be seen from equations

\[
P_1 = U_1 I_1 = G_{12} U_1 U_2 \quad \text{and} \quad P_2 = U_2 I_2 = G_{21} U_2 U_1
\]

that for a lossless reciprocal network the sign of the transfer conductances determines the direction in which the mean power flows from one port to the other. Not only can the direction of the flow be controlled by \( T_D \) but also the magnitude of the transferred mean power. For example, if \( T_D = 0 \) or \( T/2 \), there is no mean power transfer from one port to the other. A net power transfer takes place if \( T_D \in (0, T/2) \) or \( T_D \in (T/2, T) \). If \( T_D \) belongs to the first half period, the mean power is transferred from the input port to the output port, whereas for \( T_D \) belonging to the second half period results in an opposite power transfer direction. It is conjectured that for a given network, the maximum mean power transferred from one port to the other, occurs when \( T_D = T/4 \), respectively \( TD = T/2 + T/4 = 3T/4 \). Consequently, controlling the time delay \( T_D \) represents an efficient modality to manage the power transfer between two energy reservoirs. In this case the reciprocal network may consist from just one reactive element – a series inductance or a parallel capacitance – as in [5]. In some applications the reciprocal network may be replaced by a \( \lambda/4 \) transmission line [6]. This is a predictable result as a transmission line can be approximated by a cascade of \( \Gamma \) or \( \Pi LC \) two-ports.

IV. DC-DC CONVERTERS WITH FORCED GYRATOR-LIKE BEHAVIOR

By making use of a sliding-mode control, some dc-dc converters can be forced to show a dc gyrator-like behavior [11]. This idea will be illustrated with reference to the ideal buck converter shown in Fig.6, where a continuous-conduction mode will be assumed. As can be seen, the standard buck topology has been supplemented with a \( LC \) input filter in order to reduce the \( EMI \) level. In the state-space this circuit represents a 4-th order dynamical system, the state vector being \( \mathbf{x} = [i_L, i_C, u_C, u_L] \). Kirchhoff’s laws and constitutive relations yield the state equations of the system:
The fixed point $x^\star$ of this dynamic system, which corresponds to the steady-state regime, results by equating to zero the right sides of eqs. (24)-(26):

$$
x^\star = \left[ \begin{array}{c} i_{L1}^\star \\ i_{L2}^\star \\ u_{C1}^\star \\ u_{C2}^\star \end{array} \right] = \left[ \begin{array}{c} g^2R U_s \\ g U_s \\ U_s \\ g R U_s \end{array} \right] .
$$

From the port conditions

$$
i_1 = i_{L1}, \quad u_1 = U_s, \quad u_2 = u_{C2}, \quad i_2 = u_{C2}/R \quad (28)
$$

and the control condition $i_{L2} = g U_s$, it follows for the steady-state regime that

$$
i_1 = g U_s, \quad i_2 = g U_1. \quad (29)
$$

These equations correspond to an ideal dc gyrator; the missing "-" sign in the second equation is due to the active sign rule chosen at this time for the output port (Fig.6). It should be noted that as $0<\sigma<1$, from eqs. (23) and (27) follows that the gyration conductance is related to the load resistance by the condition $g R < 1$.

To conclude the analysis we have to check the stability of the fixed point, that is of the steady-state behavior of the dc gyrator. To this purpose we have to compute the Jacobian matrix of the nonlinear dynamical system (24)-(26):

$$
J(x^\star) = \begin{bmatrix}
\frac{\partial f_1}{\partial i_{L1}} & \frac{\partial f_1}{\partial u_{C1}} & \frac{\partial f_1}{\partial u_{C2}} \\
\frac{\partial f_2}{\partial i_{L1}} & \frac{\partial f_2}{\partial u_{C1}} & \frac{\partial f_2}{\partial u_{C2}} \\
\frac{\partial f_3}{\partial i_{L1}} & \frac{\partial f_3}{\partial u_{C1}} & \frac{\partial f_3}{\partial u_{C2}} \\
\frac{\partial f_4}{\partial i_{L1}} & \frac{\partial f_4}{\partial u_{C1}} & \frac{\partial f_4}{\partial u_{C2}}
\end{bmatrix},
$$

where $f_1$, $f_2$ and $f_3$ are the corresponding right sides of eqs. (24)-(26). Solving the characteristic equation

$$
\det(x I - J(x^\star)) = 0,
$$

yields the eigenvalues

$$
\lambda_1 = -\frac{1}{RC_2}, \quad \lambda_{2,3} = \frac{g^2 R}{2C_1} \pm \sqrt{\left(\frac{g^2 R}{2C_1}\right)^2 - \frac{1}{L_1 C_1}}. \quad (32)
$$

The first eigenvalue assures the exponential stabilization of $u_{C2}$, as noted before. The remaining two are positive real numbers if

$$
g^2 > \frac{2}{R} \sqrt{\frac{C_1}{L_1}}, \quad (32)
$$

respectively complex numbers with positive real parts, for the opposite case; in each case the fixed point is unstable. The stabilization of the fixed point can be made either inserting a compensation network in the feedback loop or adding a damping network to the $L_1 C_1 U_1$ loop.

Due to eq. (25), this is a nonlinear dynamical system. It is interesting to note, as equation (26) shows, that the dynamics of $u_{C2}$ is not related to the rest of the state variables – it increases exponentially with a time constant $RC_2$ towards the steady-state value $g R U_s$. 

where $\sigma(t)$ is a discrete switching variable: $\sigma(t)=1$ if $t \in T_{on}$ (Sw closed), respectively $\sigma(t)=0$ if $t \in T_{off}$ (Sw opened).

If we choose the switching surface for sliding-mode control as $S = i_{L2} - gu_2$ (u2=U2), and impose the stationarity conditions $S=0$ and $dS/dt = 0$, then eq. (20) yields

$$
i_{L2} = g U_s, \quad \sigma = \frac{u_{C2}}{u_{C1}}. \quad (23)
$$

Substituting eq. (23) in eqs. (19), (21) and (22) eventually leads to the converter dynamics under sliding-mode control:

$$
\frac{di_{L1}}{dt} = -\frac{1}{L_1} u_{C1} + \frac{1}{L_1} U_s, \quad (19)
$$

$$
\frac{du_{C1}}{dt} = \frac{1}{C_1} i_{L1} - \frac{1}{C_1} u_{C2} \frac{1}{u_{C1}} g U_s, \quad (21)
$$

$$
\frac{du_{C2}}{dt} = \frac{1}{C_2} g U_s - \frac{1}{RC_2} u_{C2}. \quad (25)
$$

Fig.6. The buck converter with sliding-mode control feedback

$\frac{du_{C1}}{dt} = \frac{1}{C_1} i_{L1} - \frac{1}{C_1} u_{C2} \frac{1}{u_{C1}} g U_s, \quad (21)
A simple solution for stabilizing the fixed point consists from connecting a series $R_cC_d$ circuit in derivation with the capacitor $C_1$. In steady-state operation, due to the capacitor $C_d$, this branch is interrupted and therefore the fixed point would be the same as that of the original circuit.

The resulting 4-th order dynamical system also has the $\lambda_1 = -1/RC_2$ eigenvalue; the remaining three are the solutions of the equation

$$\lambda^3 + \frac{C_1 + C_d - g^2 R_d C_d}{R_d C_d C_1} \lambda^2 + \left( \frac{1}{L_1 C_1} - \frac{g^2 R}{R_d C_d C_1} \right) \lambda + \frac{1}{R_d C_d L_1 C_1} = 0$$

(33)

According to Routh-Hurwitz theorem, the necessary and sufficient condition for this equation to have negative real part solutions is that the principal minors of the associated Hurwitz matrix should be positive [12]:

$$b_1 > 0, \quad b_2 > 0, \quad b_1b_2 > b_3,$$

(34)

where $b_1, b_2, b_3$ are the coefficients of $\lambda^2$ to $\lambda^0$, in this order. From eqs.(34) appropriate values for the damping RC circuit can be chosen.

To assess the gyrator-like behavior of the sliding mode controlled modified buck converter, a Simulink/SimPowerSystems model has been setup, as shown in Fig.7, where $U_1=20\,V,\,R=1\,\Omega$ and $g=0.5$. The simulation results are shown in Fig.8. As expected, the steady-state value of the output current is $gU_1=10\,A$.

Sliding-mode control can be used to force other topologies, like boost or cuk, to behave like a dc gyrator [11].

A possible application of gyrator-configured dc switch converters, where the controlled current source output nature of gyrators may be advantageous, is in the management of distributed power systems [11]. In this case, paralleling $n$ gyrator-like dc converters simply means the addition of the output currents $g_kU_{sk} (k=1, 2, ..., n)$ of the individual converters, as shown in Fig.9. However, for each individual gyrator to evolve along the imposed sliding surface, the corresponding control variable $\sigma$ must be in the $[0, 1]$ interval, which yields the constrain

$$0 < \frac{g_1U_{sk_1} + g_2U_{sk_2} + ... + g_nU_{skn}}{U_{sk}} < 1.$$  

(35)

If all the input sources have the same value, as well as all gyration conductances, rel.(35) becomes

$$g < \frac{1}{nR}.$$  

(36)
Different distribution strategies for the currents are possible due to the fact that each output current is independently controlled [11].

Although different converter topologies can be used for paralleling purposes, the buck gyrator-like converter seems to be the best choice, as demonstrated in [11].

V. CONCLUSIONS

The paper analyses two types of gyrator-like dc-dc converters which are used in dc power management. The first one, which has an unforced dc gyrator-like behavior, is operated at constant frequency. The dc power transfer from one port to the other can be simply and efficiently controlled by varying the time delay between the switched bridges. The gyrator-like behavior of the second converter type is forced with an appropriate sliding-mode control. This converter type is suited for applications implying paralleling of converters in distributed power systems, voltage regulation and impedance matching.

REFERENCES